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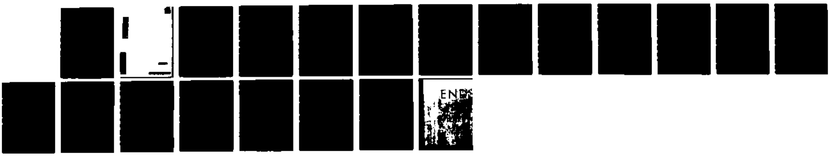

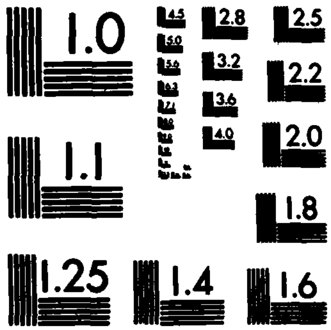

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DISTRIBUTED WIENER-POISSON CONTROL

By

Howard Weiner

TECHNICAL REPORT NO. 325

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STANFORD, CALIFORNIA

Distributed Wiener-Poisson Control

by Howard Weiner

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1. Introduction. Let $W(t)$, $t \geq 0$, $W(0) = 0$ be a standard Wiener process, independent of $N(t)$, $t \geq 0$, $N(0) = 0$ a Poisson process with (constant) unit jumps, and $EN(t) = \lambda t$, $t \geq 0$. Let their sigma fields be $F(t) = \sigma(W(s), 0 \leq s \leq t)$ and $G(t) = \sigma(N(s), 0 \leq s \leq t)$. Let $g(y)$ be a function and $L(g(y))$ a differential operator on g, for example,

$$L(g(y)) = a(y) \frac{\partial^2}{\partial y^2} g(y) + d(y) \frac{\partial}{\partial y} g(y) + c(y)g(y).$$

Let $X(t,y)$ be a stochastic process depending on $t \geq 0$ and a parameter y, such that (for $\frac{\partial X}{\partial t} \equiv X_t$),

$$X_t(t,y) = L(X(t,y)) + u(X(t,y),y) + \frac{dW(t)}{dt} + \frac{dN(t)}{dt} \tag{1.1}$$

$$X(0,y) = g(y) \equiv x.$$

with $\frac{dW(t)}{dt}$ white noise, $\frac{dN(t)}{dt}$ incremental Poisson jump process and where $u(X(t,y),y)$ is measurable with respect to $\sigma(F(t) \cup G(t))$ (i.e. u is non-anticipative) and satisfies, for A a constant, and $B > 0$ a constant,

$$|u-A| \leq B, \tag{1.2}$$

all $0 \leq t \leq T$, $0 < T \leq \infty$ a constant. The cost function for a given u satisfying (1.2) is, for $\alpha > 0$ a constant,

$$J(u,y) = \int_0^T e^{-\alpha s} E(X^2(u,s))ds. \tag{1.3}$$

The object is to characterize the optimal u for which J is minimized. The cases $T < \infty$ and $T = \infty$ are treated separately. The method employs a suitable Bellman equation, a maximum principle for parabolic partial differential-difference equations and the Ito rule. The method follows [4].

2. Finite Interval Control.

Let $T < \infty$. Define, for $0 \leq t \leq T$,

$$(2.1) \qquad V \equiv V(x,t,y) = \inf_{|u-A| \leq B} \int_0^t e^{-\alpha s} E(X^2(s,y))ds$$

and $X(0,y) = g(y) = x$.

By writing $\int_0^t = \int_0^h + \int_h^{t+h} - \int_t^{t+h}$, heuristic arguments ([2], pp. 179-180) yield a Bellman equation

$$(2.2) \qquad x^2 + \inf_{|u-A| \leq B} (uV_x) + L(g(y))V_x + \frac{1}{2} V_{xx} - \alpha V - V_t$$

$$+ \lambda(V(x+1,t,y) - V(x,t,y)) = 0,$$

with $u = u(x)$.

On heuristic grounds, a solution to (2.2) is sought such that

$$(2.3) \qquad x^2 + [(A-B) + L(g(y))]V_x + \frac{1}{2} V_{xx} - \alpha V - V_t$$

$$+ \lambda(V(x+1,t,y) - V(x,t,y)) = 0$$

for $V_x > 0$, $x > b(t,y)$

and

$$(2.4) \qquad x^2 + [(A+B) + L(g(y))]V_x + \frac{1}{2} V_{xx} - \alpha V - V_t$$

$$+ \lambda(V(x+1,t,y) - V(x,t,y)) = 0$$

for $V_x < 0$, $x < b(t,y)$

The object is to characterize the optimal u for which J is minimized. The cases $T < \infty$ and $T = \infty$ are treated separately. The method employs a suitable Bellman equation, a maximum principle for parabolic partial differential-difference equations and the Ito rule. The method follows [4].

2. Finite Interval Control.

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$$V \equiv V(x,t,y) = \inf_{|u-A| \leq B} \int_0^t e^{-\alpha s} E(X^2(s,y))ds \tag{2.1}$$

and $X(0,y) = g(y) = x$.

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$$+ \lambda(V(x+1,t,y) - V(x,t,y)) = 0$$

for $V_x < 0$, $x < b(t,y)$

where $b(t,y)$ is obtained from the following conditions, letting $V \equiv V_1$ in (2.3) and $V \equiv V_2$ in (2.4) for $0 \le t \le T$, all y:

$$V_1(b(t,y),t,y) = V_2(b(t,y),t,y)$$

$$V_{1,x}(b(t,y),t,y) = V_{2,x}(b(t,y),t,y) = 0$$

(2.5) $$V_1(x,0,y) = V_2(x,0,y) = 0.$$

(2.5a) $$V_{1,xx}(b(t,y),t,y) \ge 0 .$$

For R constant, denote

(2.6) $$J(x,t,y,R) \equiv \int_0^t e^{-\alpha s} E((R+L(g(y)))s + W(s) + N(s) + x)^2 ds.$$

A direct computation verifies that $J(x,t,y,A-B)$ is a particular solution to (2.3) and $J(x,t,y, A+B)$ is a particular solution to (2.4).

<u>Assumption 1</u>. There is a non-zero solution $\bar{H}_1(x,t,y)$ with $H_1(x,0,y) = 0$ to

$$-\alpha H + (A-B + L(g(y)))H_x + \frac{1}{2} H_{xx} - H_t + \lambda(H(x+1,t,y) - H(x,t,y)) = 0$$

such that

$$H_1(x,t,y) = O(e^{-\beta x})$$

(2.7) $$H_{1,xx}(x,t,y) = O(e^{-\delta x})$$

for some $\beta > 0$, $\delta > 0$, all t,y, as $x \to \infty$.

Also, there is a non-zero solution $H_2(x,t,y)$ with $H_2(x,0,y) = 0$, to

$$-\alpha H + (A+B + L(g(y)))H_x + \frac{1}{2} H_{xx} - H_t$$

$$+ \lambda(H(x+1,t,y) - H(x,t,y)) = 0$$

such that

$$H_2(x,t,y) = O(e^{\delta x})$$

$$(2.8) \qquad H_{2,xx}(x,t,y) = O(e^{\lambda x})$$

for some $\delta > 0$, $\lambda > 0$, all t,y, as $x \to -\infty$.

One may then write

$$(2.9) \qquad V_1(x,t,y) = J(x,t,y,\ A-B) + H_1\ (x,t,y)$$

$$(2.10) \qquad V_2(x,t,y) = J(x,t,y,A+B) + H_2(x,t,y)$$

and assume $b(t,y)$ is determined from (2.5), (2.9), (2.10).

This motivates

<u>Theorem 1</u> Assume the conditions and results of sections 1,2 hold for $0 < T < \infty$.

The optimal u_0 may be expressed as

$$(2.11) \qquad u_0(X_0(t,y),y) = \begin{cases} A-B \text{ if } X_0(t,y) > b(T-t,y) \\ \\ A+B \text{ if } X_0(t,y) \le b(T-t,y) \end{cases}$$

where $b(t,y)$ is obtained from (2.5), (2.9)-(2.10) and

$$(2.12) \qquad X_{0t}(t,y) = L(X_0(t,y)) + u_0\ (X_0(t,y),y) + \frac{dW(t)}{dt} + \frac{dN(t)}{dt}$$

with $X_0(0,y) = g(y) \equiv x$.

<u>Proof</u> Let $D \equiv V_{xx}$.

From (2.3), (2.4) (omitting (t,y) arguments)

$$(2.13) \qquad (A\underline{+}B + L(g(y)))\ D_x + \frac{1}{2}\,D_{xx} - \alpha D - D_t - \lambda D = -2\ -\lambda D(x+1)$$

where the argument on the left side of (2.13) is x.

From (2.6),

(2.14) $J_{xx}(x,t,y, A\underline{+}B) > 0.$

Assumption 1, (2.7)-(2.10), (2.14), imply that

(2.15) $V_{1,xx}(x,t,y) > 0$, for each t,y, as $x \to \infty$

and

(2.16) $V_{2,xx}(x,t,y) > 0$, for each t,y, as $x \to -\infty$.

For fixed (t,y), (omitting the fixed arguments (t,y)), suppose there existed a finite number $r > b$, and a number β, $0 < \beta < 1$ and such that using (2.5a),

(2.17) $D(x) < 0, \quad b < r - \beta < x < r, \quad D(r) = 0,$

and

(2.17) $D(x) > 0, \quad x > r,$

using (2.15). It follows that for $x \geq r-\beta$, the left side of (2.13) is negative. By ([1], Lemma 1, p.34), D cannot have a negative minimum for $x \geq r-\beta$, a contradiction to (2.17), since (2.15) and (2.5a) hold. Hence $D \geq 0$ for $x \geq b$.

For $b-1 \leq x < b$, a similar argument using (2.16) and that $D(x) > 0$ for $b \leq x < b + 1$ yields that $D(x) \geq 0$ for $b-1 \leq x < b$. Continuing by iteration, $D(x) > 0$ for $x < b$, so that $D \geq 0$ for all (x,t,y).

Also, $V_x(b(t)) = 0$ by (2.5), implying that (2.3),(2.4) (2.9), (2.10) is a solution to the Bellman equation (2.2). To show that u_0 is optimal, define

(2.19) $K(X(t,y),t,y) \equiv V(X(t,y),T-t,y)e^{-\alpha t}$

for $0 \leq t \leq T$.

Noting that $K(X(0,y),0,y) = V(x,T,y)$ and $K(X(T,y),T,y) = 0$, the Ito rule yields that ([2], pp. 125-126) for a u and corresponding $X(t,y)$,

$$(2.20)\quad \int_0^T e^{-\alpha s} X^2(s,y)ds - V(x,T,y) =$$

$$\int_0^T e^{-\alpha s}(-\alpha V(X(s,y),T-s,y) - V_t(X(s,y),T-s,y)$$

$$+ \inf_{|u-A| \le B} (u(X(s,y),y) \quad V_x(X(s,y),T-s,y))$$

$$+ \frac{1}{2} \quad V_{xx}(X(s,y),T-s,y) + L(g(y)) \; V_x(X(s,y),T-s,y) + X^2(s,y))ds$$

$$+ \int_0^T e^{-\alpha s}(V(X(s,y),T-s,y)dN(s)$$

$$+ \int_0^T e^{-\alpha s} V_x(X(s,y),T-s,y)dW(s)$$

$$+ \int_0^T e^{-\alpha s}(u(X(s,y),y)V_x(X(s,y),T-s,y) - \inf_{|u-A| \le B} (u(X(s,y),y)V_x(X(s,y),T-s,y))d$$

The fourth integral on the right of (2.20) is non-negative. Upon taking expectations of (2.20), the third integral on the right is zero, and one obtains (omitting the arguments from the first and fourth integrals, and combining the first and second integrals, all on the right)

$$(2.21)\quad \int_0^T e^{-\alpha s} E(X^2(s,y))ds - V(x,T,y) =$$

$$E \int_0^T e^{-\alpha s}(-\alpha V - V_t + \inf_{|u-A| \le B} (uV_x) + L(g(y))V_x + \frac{1}{2} V_{xx} + X^2$$

$$+ \lambda(V(X(s)+1)-V)ds$$

$$+ E \int_0^T e^{-\alpha s}(uV_x - \inf_{|u-A| \le B} (uV_x))ds.$$

The first integral on the right of (2.21) is zero by (2.2)-(2.4), and

the second integral is non-negative, and is zero for $u = u_0$, $X = X_0$. Hence (2.21) yields that

$$\int_0^T e^{-\alpha s} E(X^2(s,y))ds \geq V(x,T,y) \tag{2.22}$$

and

$$\int_0^T e^{-\alpha s} E(X_0^2(s,y))ds = V(x,T,y), \tag{2.23}$$

and (2.22), (2.23) imply u_0 is optimal.

3. Infinite Interval Control

Assume the conditions of section 1 hold for $T = \infty$ so that (1.3) is now

$$J(u,y) = \int_0^\infty e^{-\alpha s} E(X^2(s,y))ds. \tag{3.1}$$

Define

$$V \equiv V(x,y) = \inf_{|u-A| \leq B} \int_0^\infty e^{-\alpha s} E(X^2(s,y))ds \tag{3.2}$$

$$\text{with } X(0,y) = g(y) \equiv x.$$

By writing $\int_0^\infty = \int_0^h + \int_h^\infty$, heuristic arguments ([2], pp. 179-180) yield a Bellman equation

$$\begin{aligned} x^2 - \alpha V(x,y) + \frac{1}{2} V_{xx}(x,y) + \inf_{|u-A| \leq B} (u(x,y)V_x(x,y)) \\ + L(g(y))V_x(x,y) + \lambda(V(x+1,y) - V(x,y)) = 0. \end{aligned} \tag{3.3}$$

On heuristic grounds, a solution to (3.3) is sought such that (omitting the (x,y) arguments)

$$(3.4) \qquad x^2 - \alpha V + \frac{1}{2} V_{xx} + (A-B + L(g(y)))V_x + \lambda(V(x+1,y)-V) = 0$$

for $V_x > 0$, $x > b(y)$

and

$$(3.5) \qquad x^2 - \alpha V + \frac{1}{2} V_{xx} + (A+B+L(g(y))V_x + \lambda(V(x+1,y)-V) = 0$$

for $V_x < 0$, $x < b(y)$,

where $b(y)$ is obtained from the following matching conditions, letting $V \equiv V_1$ in(3.4) and $V \equiv V_2$ in (3.5):

$$V_1(b(y),y) = V_2(b(y),y)$$

$$(3.6) \qquad V_{1,x}(b(y),y) = V_{2,x}(b(y),y) = 0$$

$$(3.6a) \qquad V_{1,xx}(b,y),y) \geq 0 \; .$$

For R constant, denote

$$(3.7) \qquad J(x,y,R) = \int_0^\infty e^{-\alpha s} E((R+L(g(y)))s + W(s)+N(s)+x)^2 ds.$$

A computation verifies that $J(x,y,A-B)$ is a particular solution to (3.4) and $J(x,y, A+B)$ is a particular solution to (3.5).

<u>Assumption 2.</u> There is a non-zero solution $H_1(x,y)$ to (omitting (x,y) argument)

$$-\alpha H + (A-B + L(g(y)))H_x + \frac{1}{2} H_{xx} + \lambda(H(x+1,y)-H) = 0$$

such that

$$H_1(x,y) = O(e^{-\beta x})$$

$$(3.8) \qquad H_{1,xx}(x,y) = O(e^{-\delta x})$$

for some $\beta > 0$, $\delta > 0$, for each y, as $x \to \infty$. Similarly, there is a non-zero solution $H_2(x,y)$ to

$$-\alpha H + (A+B + L(g(y)))H_x + \frac{1}{2} H_{xx} + \lambda(H(x+1,y)-H) = 0$$

such that

$$H_2(x,y) = O(e^{\delta x})$$

$$H_{2,xx}(x,y) = O(e^{\lambda x}) \tag{3.9}$$

for some $\delta > 0$, $\lambda > 0$, for each y as $x \to -\infty$.

Then one may write

$$V_1(x,y) = J(x,y,A-B) + H_1(x,y) \tag{3.10}$$

$$V_2(x,y) = J(x,y,A+B) + H_2(x,y) \tag{3.11}$$

and assume that b(y) is determined from (3.6), (3.10), (3.11).

As in Section 2, one then obtains

<u>Theorem 2</u>. Assume that the conditions and results of sections 1,3 hold for $T = \infty$,

The optimal u_1 may be expressed as

$$u_1(X_1(t,y),y) = \begin{cases} A-B & \text{if } X_1(t,y) > b(y) \\ \\ A+B & \text{if } X_1(t,y) \leq b(y) \end{cases} \tag{3.12}$$

where $b(y)$ is obtained from (3.6), (3.10)-(3.11) and

$$(3.13) \qquad X_{1t}(t,y) = L(X_1(t,y)) + \frac{dW(t)}{dt} + \frac{dN(t)}{dt}.$$

and $\quad X_1(0,y) = g(y) \equiv x.$

<u>Proof.</u> The proof follows that of Theorem 1.

Let $D = V_{xx}$. From (3.4), (3.5) (omitting (x,y) arguments)

$$(3.14) \quad (A\underline{+}B + L(g(y)))D_x + \frac{1}{2}D_{xx} - \alpha D - \lambda D = -2 - \lambda D(x+1)$$

where the argument on the left side of (3.14) is x.

From (3.7),

$$(3.15) \qquad J_{xx}(x,y,A\underline{+}B) > 0.$$

Assumption 2, (3.8)-(3.11), (3.15) imply that

$$(3.16) \qquad V_{1,xx}(x,y) > 0 \quad \text{for each } t,y, \text{ as } x \to \infty$$

and

$$(3.17) \qquad V_{2,xx}(x,y) > 0 \quad \text{for each } t,y, \text{ as } x \to -\infty.$$

By an argument identical to that given in the proof of Theorem 1, using the appropriate maximum principle ([1], Theorem 18, p. 53), it follows that $D(x,y) \geq 0$ for all (x,y). Since $V_x(b(y),y) = 0$

by (3.6), it follows that (3.4), (3.5), (3.10), (3.11) constitute a solution to the Bellman equation (3.3). To show that u_1 is optimal, define

$$K(X(t,y),y) = V(X(t,y),y)e^{-\alpha t} \tag{3.20}$$

for $t \geq 0$.

Noting that $K(X(0,y),y) = V(x,y)$, one obtains from the Ito rule ([2], pp. 125-126) for a u and corresponding $X(t,y)$,(omitting $(X(s,y),y)$ arguments on the right side),

$$\begin{aligned}
&\int_0^t e^{-\alpha s} X^2(s,y)ds + V(X(t,y),y)e^{-\alpha t} - V(x,y) = \\
&\int_0^t e^{-\alpha s}(X^2(s,y) - \alpha V + \inf_{|u-A| \leq B}(uV_x) + L(g(y))V_x + \tfrac{1}{2} V_{xx})ds \\
&+\int_0^t e^{-\alpha s}(uV_x - \inf_{|u-A| \leq B}(uV_x))ds \\
&+\int_0^t e^{-\alpha s}(V(X(s,y),s)dN(s) \\
&+\int_0^t e^{-\alpha s}V_x \, dW(s).
\end{aligned} \tag{3.21}$$

The second integral on the right of (3.21) is non-negative. Upon taking expectations in (3.21), the fourth term on the right is zero, and combining the first and third terms on the right yields that

$$(3.22)\qquad \int_0^t e^{-\alpha s}E(X^2(s,y))ds + E(V(X(t,y),y))e^{-\alpha t}-V(x,y) =$$

$$E\int_0^t e^{-\alpha s}(X^2(s,y)-\alpha V + \inf_{|u-A|\le B}(uV_x) + L(g(y))V_x + \frac{1}{2}V_{xx}$$

$$+\lambda(V(X(s)+1,y)-V)ds$$

$$+E\int_0^t e^{-\alpha s}(uV_x - \inf_{|u-A|\le B}(uV_x))ds.$$

By (3.3), the first term on the right of (3.22) is zero.

The second term on the right of (3.22) is non-negative, and is zero for $u = u_1$. By Assumption 2, (3.7)-(3.11), it follows that, for (x,y) fixed, there is a constant $D > 0$ such that

$$(3.23)\quad E(V(X(t,y),y))e^{-\alpha t} \le E(|x|+(|A|+B+|L(g(y)|)t+|W(t)|+N(t))^2e^{-\alpha t}+De^{-\alpha t}$$

or

$$(3.24)\quad E(V(X(t,y),y)e^{-\alpha t} \le C(x,y)t^2e^{-\alpha t}$$

for some positive constant $C(x,y)$.

Letting $t \to \infty$ in (3.22), by (3.24) one obtains

$$(3.25)\qquad \int_0^\infty e^{-\alpha s}E(X^2(s,y))ds \ge V(x,y)$$

and

$$(3.26)\qquad \int_0^\infty e^{-\alpha s}E(X_1^2(s,y))ds = V(x,y),$$

so that (3.25), (3.26) imply that u_1 is optimal.

4. Additional Constraints

Certain additional constraints may be incorporated and treated by these methods. An illustrative example in the case $T < \infty$ is the added constraint

$$E(X^2(a)) = C \tag{4.1}$$

where $0 < a \leq T$ and $C > 0$. This may be handled by adding the condition

$$V_t(x,t,y)\big|_{t=a} = e^{-\alpha a}C \tag{4.2}$$

to conditions (2.5), and proceeding as before. See [3] for another approach.

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#325

A one-dimensional Wiener plus independent Poisson control problem with state governed by a partial differential equation has integrated discounted quadratic cost function and asymmetric bounds on the control, which is a function of the current state. A Bellman equation and maximum principle for partial differential equations are used to obtain the optimal closed loop control in bang-bang form. The finite and infinite integral quadratic cost functions are treated separately.

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